Poincaré Surface of Section

- Calculate trajectory: $\theta(t)$, $\phi(t)$, $p_\theta(t)$, $p_\phi(t)$.
- Select points with $p_\phi = 0$, $\dot{p}_\phi > 0$.
- Project these points onto a plane in $(\theta, \phi, p_\theta)$-space.

Integrable system:

Invariant torus specified by two actions $J_1, J_2$.
Two angle coordinates, $\vartheta_1(t) = \omega_1(J_1, J_2) t + \vartheta_1^0$, $\vartheta_2(t) = \omega_2(J_1, J_2) t + \vartheta_2^0$,
describe winding course of trajectory around torus.
Periodic trajectories: rational $\omega_1/\omega_2$.
Quasiperiodic trajectories: irrational $\omega_1/\omega_2$.

Nonintegrable system:

Only irrational tori exist. Space between intact tori filled by periodic trajectories and chaotic trajectories.