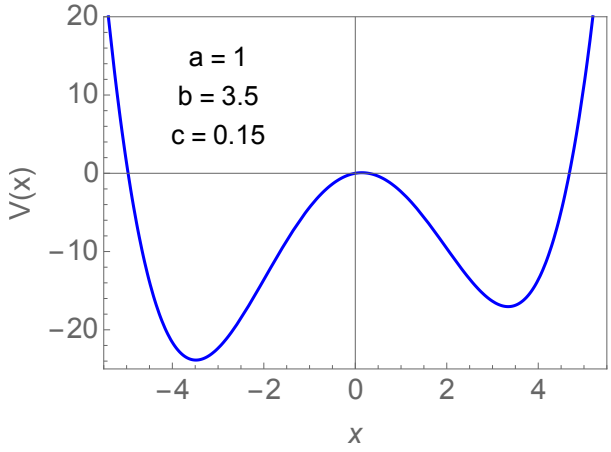
Velocity Vector Field in Phase Plane [mln109]

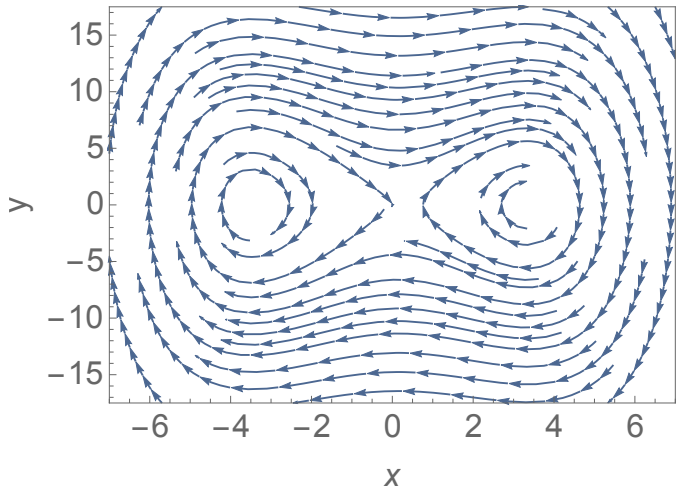
Phase portraits are readily produced computationally. We use the StreamPlot command of Mathematica for a demonstration. Consider a particle of unit mass moving in a quartic potential,

$$V(x) = ax - bx^2 + cx^4.$$

For the parameter values indicated, we have an asymmetric double-well potential as shown. The presence of three fixed points is readily recognized. Zooming into their vicinity reveals their nature.

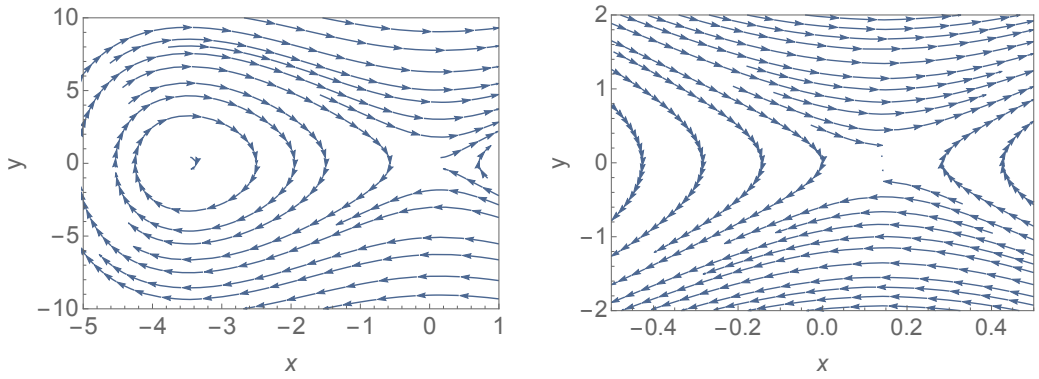


The components of the velocity vector field are $\dot{x} = y$ and $m\dot{y} = -dV/dx$. The presence of three fixed points is readily recognized.



[over]

Zooming into their vicinity reveals their nature.



Adding damping changes the force to $F = -dV/dx - \gamma\dot{x}$. We use $\gamma = 0.5$. The hyperbolic fixed point is still present. The two elliptic fixed point turn into spiral attractors.