Conservation Laws [mln2]

Single Particle

- The *linear momentum* $\mathbf{p} = m\mathbf{v}$ in adirection (specified by vector $\mathbf{s}$) in which the applied force $\mathbf{F}$ vanishes is a constant in time:

$$\dot{\mathbf{p}} \cdot \mathbf{s} = \mathbf{F} \cdot \mathbf{s}.$$

- The *angular momentum* $\mathbf{L} = m(\mathbf{r} \times \mathbf{v})$ is a constant in time if the applied force $\mathbf{F}$ exerts zero torque $\mathbf{N}$:

$$\dot{\mathbf{L}} \doteq \frac{d}{dt}(\mathbf{r} \times \mathbf{p}) = \underbrace{\dot{\mathbf{r}} \times \mathbf{p}}_{0} + \mathbf{r} \times \dot{\mathbf{p}} = \mathbf{r} \times \mathbf{F} = \mathbf{N}.$$

- If the applied force $\mathbf{F}$ is *conservative*, then the *total energy* E, which is the sum of the *kinetic energy* T and *potential energy* V, is a constant in time:

$$E = T + V; \qquad T = \frac{1}{2}mv^2, \quad V(\mathbf{r}) = -\int_{\mathbf{r}_0}^{\mathbf{r}} \mathbf{F} \cdot d\mathbf{s}, \quad \mathbf{F}(\mathbf{r}) = -\nabla V(\mathbf{r}).$$

System of Particles

External force: $\mathbf{F}^{(e)} = \sum_i \mathbf{F}_i^{(e)}$. Internal forces: $\mathbf{F}_{ij} = -\mathbf{F}_{ji}$ with $\mathbf{F}_{ij} \| \mathbf{r}_{ij}$.

- The component of *total linear momentum* $\mathbf{p}$ in a direction in which the *external force* $\mathbf{F}^{(e)}$ vanishes is a constant in time:

$$\dot{\mathbf{p}} \cdot \mathbf{s} \doteq \sum_i \dot{\mathbf{p}}_i \cdot \mathbf{s} = \mathbf{F}^{(e)} \cdot \mathbf{s} + \underbrace{\sum_{ij} \mathbf{F}_{ij} \cdot \mathbf{s}}_{0}.$$

- The *total angular momentum* $\mathbf{L}$ is a constant in time if the external force $\mathbf{F}^{(e)}$ exerts zero torque $\mathbf{N}^{(e)}$:

$$\dot{\mathbf{L}} \doteq \sum_i \frac{d}{dt}(\mathbf{r}_i \times \mathbf{p}_i) = \sum_i \mathbf{r}_i \times \mathbf{F}^{(e)} = \mathbf{N}^{(e)}.$$

- If the forces $\mathbf{F}^{(e)}$ and $\mathbf{F}_{ij}$ are *conservative*, then the *total (mechanical) energy* E of the system is a constant in time:

$$E = T + V; \qquad T = \sum_i \frac{1}{2}m_i v_i^2, \quad V = \sum_i V_i^{(e)} + \sum_{i<j} V_{ij}.$$

Non-conservative forces (friction, attenuation) imply energy dissipation. Some mechanical energy is then converted into thermal energy or radiation.