Isoclines

Isoclines are a simple device used to identify, in conjunction with the analysis of fixed points, all salient features in the 2D phase flow of a given pair of 1st order ODEs:

$$\dot{x}_1 = f_1(x_1, x_2), \qquad \dot{x}_2 = f_2(x_1, x_2).$$

Isoclines are sets of curves on which all phase-plane trajectories have tangents with one and the same direction.

Two special directions are commonly singled out:

1. Isoclines intersected *vertically* by all trajectories are determined by the curves representing the equation $f_1(x_1, x_2) = 0$.

2. Isoclines intersected *horizontally* by all trajectories are determined by the curves representing the equation $f_2(x_1, x_2) = 0$.

All points of intersection between a curve of isocline 1 and a curve of isocline 2 are fixed points of the phase flow.

Alternative isoclines, representing locations in the phase plane where all trajectories have slope ± 1 are determined by the solutions of the equations $f_1(x_1, x_2) = \pm f_2(x_1, x_2)$.