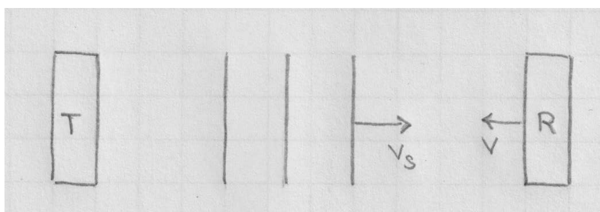
Longitudinal Doppler Effect [mln57]

Sound:

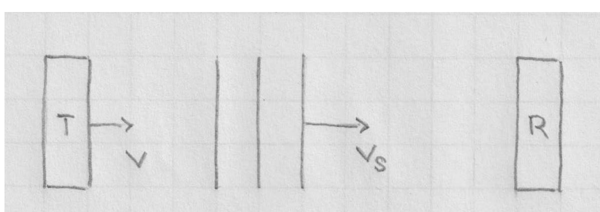
Sound wave propagates with velocity v_s through medium. Transmitter T and receiver R in motion with relative velocity $v < v_s$ toward each other.

Time interval: Δt.
Transmitter frequency: ν_T.
Number of cycles transmitted in Δt: $N_T = \nu_T \Delta t$.
Number of cycles received in Δt: $N_R = \nu_R \Delta t$.

(a) Transmitter at rest in medium:
Distance occupied by N_T cycles: $N_T \lambda = v_s \Delta t$.
Wavelength in medium: $\lambda = \dfrac{v_s}{\nu_T} = \dfrac{v_s + v}{\nu_R}$.
Frequency at receiver: $\nu_R = \dfrac{v_s + v}{\lambda} = \nu_T(1 + v/v_s)$.



(b) Receiver at rest in medium:
Distance occupied by N_T cycles: $N_T \lambda = (v_s - v)\Delta t$.
Wavelength in medium: $\lambda = \dfrac{v_s - v}{\nu_T} = \dfrac{v_s}{\nu_R}$.
Frequency at receiver: $\nu_R = \dfrac{v_s}{\lambda} = \dfrac{\nu_T}{1 - v/v_s} \overset{v \ll v_s}{\rightsquigarrow} \nu_T(1 + v/v_s)$.



Light:

Transmitter T and receiver R in motion with relative velocity v toward each other. Light wave propagates with velocity c relative to T and relative to R.

Proper time interval measured in T-frame: Δt_T.

Dilated time interval measured in R-frame: $\Delta t_R = \dfrac{\Delta t_T}{\sqrt{1 - v^2/c^2}}$.

Frequency of transmitter: ν_T.

Number of cycles transmitted in Δt_T: $N_T = \nu_T \Delta t_T$.

Distance occupied by N_T cycles in T-frame: $N_T \lambda_T = c\Delta t_T$.

Wavelength in T-frame: $\lambda_T = \dfrac{c\Delta t_T}{N_T} = \dfrac{c}{\nu_T}$.

Distance occupied by N_T cycles in R-frame: $N_T \lambda_R = (c - v)\Delta t_R$.

Wavelength in R-frame:

$$\lambda_R = \frac{c - v}{N_T}\Delta t_R = \frac{c - v}{\nu_T}\frac{\Delta t_R}{\Delta t_T} = \frac{c}{\nu_T}\frac{1 - v/c}{\sqrt{1 - v^2/c^2}} = \frac{c}{\nu_T}\sqrt{\frac{1 - v/c}{1 + v/c}}.$$

Frequency in R-frame: $\nu_R = \dfrac{c}{\lambda_R} = \nu_T \sqrt{\dfrac{1 + v/c}{1 - v/c}} \overset{v \ll c}{\leadsto} \nu_T(1 + v/c)$..