Coordinate Transformations [mln58]

Galilei transformation:

$$x' = x - vt, \quad y' = y, \quad z' = z, \quad t' = t.$$

Lorentz transformation:

$$x' = \frac{x - vt}{\sqrt{1 - v^2/c^2}}, \quad y' = y, \quad z' = z, \quad t' = \frac{t - vx/c^2}{\sqrt{1 - v^2/c^2}}.$$

- Check time dilation by clock at rest in S.
  Proper time interval: $\Delta\tau \doteq t_2 - t_1$.
  Lorentz transformation for $x_1 = x_2$: $t'_2 - t'_1 = \dfrac{t_2 - t_1}{\sqrt{1 - v^2/c^2}}$.
  Dilated time: $\Delta t' = \dfrac{\Delta\tau}{\sqrt{1 - v^2/c^2}}$.
- Check length contraction by ruler at rest in S'.
  Proper length: $\ell_0 \doteq x'_2 - x'_1$.
  Lorentz transformation for $t_1 = t_2$: $x'_2 - x'_1 = \dfrac{x_2 - x_1}{\sqrt{1 - v^2/c^2}}$.
  Contracted length: $\ell = \ell_0\sqrt{1 - v^2/c^2}$.
- Check relativity of simultaneity.
  Clocks between S and S' synchronized, $t = t' = 0$ and $x = x' = 0$.
  Reading of t' in S' at $x' = 0$: $t'(t, x) = \dfrac{t - vx/c^2}{\sqrt{1 - v^2/c^2}}$.
  Clocks in S go slower and are out of sync when viewed from S'.

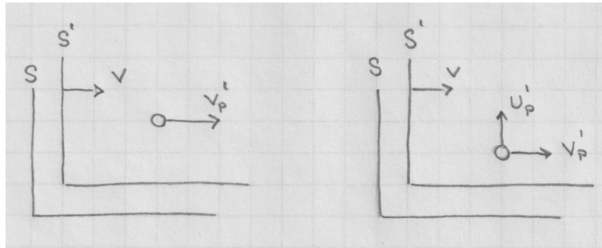
Longitudinal velocity addition:

Substitute $x = v_p t$ and $x' = v'_p t'$ into transformation equations.

- Nonrelativistic: $v_p = v'_p + v \quad \Rightarrow \quad v'_p = v_p - v$.
- Relativistic: $v'_p t' = \dfrac{v_p t - vt}{\sqrt{1 - v^2/c^2}}, \quad t' = \dfrac{t - v_p t(v/c^2)}{\sqrt{1 - v^2/c^2}}$.

$$\Rightarrow \; v'_p = \frac{v_p - v}{1 - v_p v/c^2} \quad \Rightarrow \; v_p = \frac{v'_p + v}{1 + v'_p v/c^2} < c.$$

Universality of c: $v_p \to c \quad \Rightarrow \; v'_p \to \dfrac{c - v}{1 - cv/c^2} = c$.



Transverse velocity addition:

Set $y' = u'_p t'$, $y = u_p t$ and $x = v_p t$, $x' = v'_p t'$.

Then use $y = y'$ and time dilation, $t' = \dfrac{t - v_p t(v/c^2)}{\sqrt{1 - v^2/c^2}}$.

- Nonrelativistic: $u'_p = u_p$.
- Relativistic: $u'_p = \dfrac{y'}{t'} = \dfrac{y}{t'} = \dfrac{u_p t}{t'}$.

$$\Rightarrow \; u'_p = \frac{u_p\sqrt{1 - v^2/c^2}}{1 - v_p v/c^2} \qquad \Rightarrow \; u_p = \frac{u'_p\sqrt{1 - v^2/c^2}}{1 + v'_p v/c^2}.$$