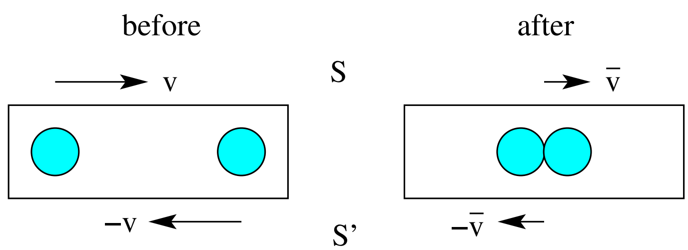
Relativistic Momentum [mln63]

Ansatz for relativistic momentum: $\mathbf{p} = m(v)\mathbf{v}$.

Two particles with equal masses m as measured when at rest are undergoing an inelastic collison as shown in the lab frame S and in the frame S' moving with velocity v to the right. Note the double role of v in this setting.



1. Relation between v (relative velocity between frames) and $\pm\bar{v}$ (particle velocity after collison) from [mln58] ($v_p = \bar{v}$ and $v'_p = -\bar{v}$):

$$v_p = \frac{v_p + v}{1 + v'_p v/c^2} \quad \Rightarrow \quad \bar{v} = \frac{-\bar{v} + v}{1 - \bar{v}v/c^2} \quad \Rightarrow \quad v = \frac{2\bar{v}}{1 + \bar{v}^2/c^2}.$$

2. Conservation of total momentum (in frame S):

$$m(v)v + m(0)0 = M(\bar{v})\bar{v}.$$

3. Lorentz invariance of momentum conservation implies [mex221]:

$$M(\bar{v}) = m(v) + m(0).$$

Relativistic mass from 1.–3. [mex222]:

$$m(v) = \frac{m_0}{\sqrt{1 - v^2/c^2}},$$

where $m_0 = m(0)$ is called the *rest mass*.

Relativistic momentum:

$$\mathbf{p} = \frac{m_0 \mathbf{v}}{\sqrt{1 - v^2/c^2}}.$$