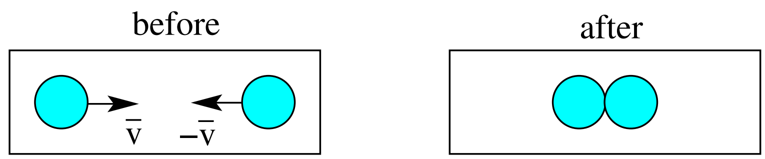
Relativistic Energy I [mln64]

The two colliding particles of equal mass viewed from the frame in which the total momentum is zero. Here all energy is carried by massive particles.



Relativistic mass is conserved in collision, a conclusion inferred from momentum conservation [mln63]:

$$M(0) = m(\bar{v}) + m(-\bar{v}) = 2m(\bar{v}) = \frac{2m_0}{\sqrt{1 - \bar{v}^2/c^2}}.$$

Increase in rest mass (after collision):

$$\Delta M_0 = M(0) - 2m_0 = 2m_0 \left(\frac{1}{\sqrt{1 - \bar{v}^2/c^2}} - 1 \right) \overset{v \ll c}{\leadsto} \frac{m_0 \bar{v}^2}{c^2}.$$

Relativistic energy (in general):

$$E \doteq m(v)c^2 = \frac{m_0 c^2}{\sqrt{1 - v^2/c^2}}.$$

Conservation of relativistic energy (in collision) follows from conservation of relativistic mass in this case, but is more general:

$$\Delta E = M(0)c^2 - 2m(\bar{v})c^2 = 0.$$

Relativistic kinetic energy (in general):

$$T \doteq E - m_0 c^2 = m_0 c^2 \left(\frac{1}{\sqrt{1 - v^2/c^2}} - 1 \right) \overset{v \ll c}{\leadsto} \frac{1}{2} m_0 v^2.$$

Kinetic energy converted into thermal energy (during collision):

$$\Delta Q = -\Delta T = \Delta M_0 c^2 \simeq 2 \left(\frac{1}{2} m_0 \bar{v}^2 \right).$$