Solution by Separation of Variables [mln72]

Velocity-dependent attenuation

Equation of motion: $m\ddot{x} = F$ with $F = f(\dot{x})g(t)$.

$$\Rightarrow\ m\frac{dv}{dt} = f(v)g(t) \quad \Rightarrow\ m\int_{v_0}^{v}\frac{dv}{f(v)} = \int_0^t dt\, g(t).$$

$$\text{Solve for } v(t). \quad \text{Then calculate } x(t) = \int_0^t dt\, v(t).$$

Applications: [mex15], [mex16], [mex230].

Rocket motion

Instantaneous momentum of rocket: $p_R(t) = m(t)v(t)$.

Momentum increment of exhaust gases: $\Delta p_E(t) = -[u - v(t)](-\Delta m)$.

Speed of exhaust gases relative to rocket: u.

Equation of motion: $\dot{p}_R + \dot{p}_E = F_{\text{ext}}$.

$$\Rightarrow\ m\dot{v} + \dot{m}v - (u - v)(-\dot{m}) = F_{\text{ext}}.$$

$$\Rightarrow\ m\dot{v} + \dot{m}u = F_{\text{ext}}.$$

Rocket motion in free space:

$$F_{\text{ext}} = 0 \quad \Rightarrow\ \frac{dv}{u} = -\frac{dm}{m} \quad v(t) = u\ln\frac{m_0}{m(t)}.$$

Applications: [mex17], [mex18], [mex229].

Photon rocket [mex223].