Challenges for Newtonian Mechanics [mln75]

Newton's second law, $\mathbf{F} = m\mathbf{a}$, relates cause and effect.

Methodological challenge: Not all causes are explicitly known prior to the solution of the problem.

Prominent among unknown causes are *forces of constraint*.

The problem is often obscured by ad-hoc ways of circumnavigation.

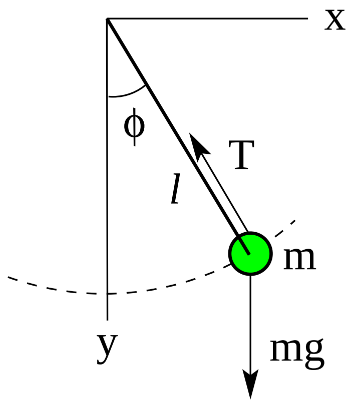
Example: Plane pendulum.

Position vector: $\mathbf{r} = (x, y)$.

Equation of motion: $m\ddot{\mathbf{r}} = m\mathbf{g} + \mathbf{T}$.

Known force: $m\mathbf{g}$ (weight).

Unknown force: $\mathbf{T}$ (tension).



Different approaches to solving the plane-pendulum problem:

1. Stick to Newtonian mechanics. This is awkward. You have to deal with four equations for four unknowns. After eliminating three of the unknowns you end up with one second-order differential equation for the remaining variable. [mex132]
2. Invoke D'Alembert's principle. This is advantageous. You arrive at the same second-order differential equation more directly. [mex134]
3. Start from Lagrangian. This is even better and more elegant. You arrive at the same second-order differential equation (the Lagrange equation) yet more directly, but you still have to solve it, which is more easily said than done.
4. Handle the constraint as learned from the previous two methods and use energy conservation. This is smart. You end up with a first-order differential equation, which is almost always preferable to one of second order. [mex146] [mex147]
5. Infer the Hamiltonian from the Lagrangian and find the canonical transformation to action-angle coordinates. This is super-elegant. It gives you deep insight into mechanics, but more work is needed to get the same solution as in the previous method. [mex200]