[mex110] Balancing a heavy object on a light rod

The equations of motion

$$\dot{x}_1 = x_2, \qquad \dot{x_2} = \frac{c_1 x_1}{L}\cos x_1 + \frac{c_2 x_2}{L}\cos x_1 + \frac{g}{L}\sin x_1$$

represent a point mass m being balanced on a rod of length L and negligible mass through lateral movement of the pivot. Here the variable x_1 represents the angle ϕ from the upright equilibrium position and the variable x_2 the associated angular velocity $\dot{\phi}$ as explained in [mln33]. (a) Analyze the nature of the fixed point at $(x_1, x_2) = (0, 0)$ for the case with zero feedback ($c_1 = c_2 = 0$). (b) Determine the conditions for the control parameters c_1, c_2 under which the the fixed point at $(x_1, x_2) = (0, 0)$ is an attractor, i.e. for which it is asymptotically stable. (c) Solve the coupled differential equations (1) by using the NDSolve option of Mathematica. Produce plots x_1 versus x_2 for trajectories that describe (i) perfect balance established and maintained, (ii) slowly lost balance, (iii) quickly lost balance, (iv) imperfect balance maintained. Discuss the relevant parameter settings for each case.

Solution: