Snell’s law for the refraction of a ray of light states that the ratio of the sines of the angles of incidence and refraction is equal to the ratio of phase velocities in the two media or, equivalently, equal to the ratio of the indices of refraction in the opposite media:

\[
\frac{\sin \theta_1}{\sin \theta_2} = \frac{v_1}{v_2} = \frac{n_2}{n_1}.
\]

A particle of mass \(m\) moving in the \(xy\)-plane is subject to a potential energy which assumes the constant value \(V_1\) at \(y \geq 0\) and the constant value \(V_2\) at \(y < 0\). Let us assume that \(V_2 < V_1\). Use conversation laws to show that if the particle approaches the \(x\)-axis with speed \(v_1\) at an angle \(\theta_1\) as shown, it will proceed with a different speed \(v_2\) at a different angle \(\theta_2\) after crossing the line where the potential energy changes abruptly. Show in particular that the relation,

\[
\frac{\sin \theta_1}{\sin \theta_2} = \frac{v_2}{v_1}, \quad v_2 = v_1 \sqrt{1 + \frac{V_1 - V_2}{K_1}}, \quad K_1 = \frac{1}{2} mv_1^2,
\]

between the two angles holds, implying an inverse relationship between velocities and angles.

Solution: