[mex193] Hamiltonian of free particle in rotating frame

Consider a particle of mass m that is free to move in the xy-plane.
(a) Find the Hamiltonian $H(r, \theta, p_r, p_\theta)$, where $x = r\cos\theta, y = r\sin\theta$.
(b) Convert the resulting canonical equations into two 2^{nd}-order ODEs for r and θ.
(c) Perform a point transformation $R = r, \phi = \theta + \omega t$ to a rotating frame with $\omega = \text{const}$. Find the transformed Hamiltonian $\tilde{H}(r, \phi, p_R, p_\phi)$ following the prescription derived in [mex80] and convert the resulting canonical equations into two 2^{nd}-order ODEs for R and ϕ.
(d) Derive the equations of motion found in (c) from those found in (b) through direct substitution of the transformation relations.

Solution: