[mex243] Classical inverse scattering problem I

The reconstruction of the (central force) scattering potential $V(r)$ from the observed scattering cross section $\sigma(\theta)$ as outlined in [mln104] involves the transformation of the orbital integral

$$\frac{\pi-\theta(s)}{2}=\int_0^{u_m}\frac{s\,du}{\sqrt{1-\frac{V(1/u)}{E}-s^2u^2}},\qquad s^2u_m^2+V(1/u_m)/E=1,$$

where $u\doteq 1/r$ and $\theta(s)$ is the scattering angle as a function of the impact parameter, into the relation

$$\pi\sqrt{\alpha}-\int_0^{\alpha}dx\,\tilde{\theta}'(x)\sqrt{\alpha-x}=\pi\int_0^{u_m(\alpha)}\frac{du}{w(u)},\qquad u_m^2=\alpha[w(u_m)]^2,$$

for the unknown function $w(u)\doteq\sqrt{1-V(1/u)/E}$, where $x\doteq 1/s^2$ and $\tilde{\theta}(x)=\theta(s)$. Carry out the initial steps as indicated in [mln104]. Integrate by parts on the left and interchange the order of integrations on the right.

Solution: