[mex266] Principal moments of square-shaped tile
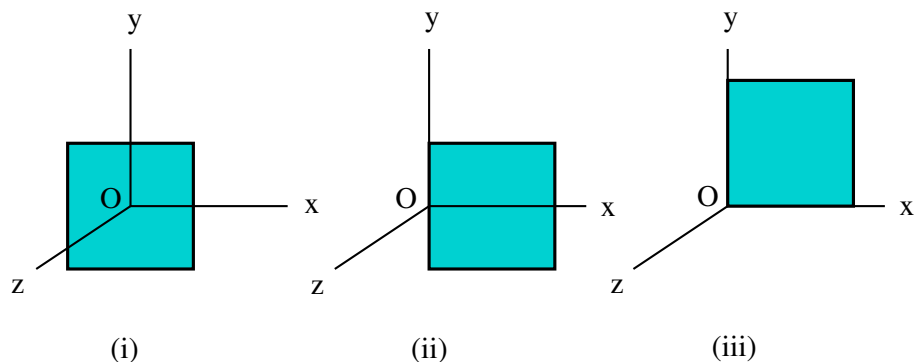
Consider a flat, square-shaped tile with sides of length d and mass $m = \sigma d^2$, where σ is the (uniform) area mass density. The tile is placed in the xy-plane with (i) the center of mass, (ii), the center of one edge, or (iii) one corner at the origin O of the coordinate system.

(a) Use symmetry arguments to determine the direction of the three principal axes for rotation about O in each of the three cases. Identify these axes in the coordinate systems of parts (i), (ii), and (iii), respectively, and label them 1, 2, 3.

(b) Determine the principal moments I_1, I_2, I_3 for cases (i)-(iii) and express them in units of md^2. Hint: Use the parallel-axis and perpendicular-axis theorems to derive some results from others and to check their consistency.

(i)   (ii)   (iii)

Solution: