Consider a hoop of mass $m$ and radius $r$ rolling without slipping down an incline.

(a) Determine the Lagrangian $L(x, \dot{x})$ of this one-degree-of-freedom system. Derive from it the Lagrange equation and its solution for initial condition $x_0 = 0, \dot{x}_0 = 0$.

(b) Determine the alternative Lagrangian $L(x, \theta, \dot{x}, \dot{\theta})$ and the holonomic constraint $f(x, \theta) = 0$ that must accompany it. Derive the associated three equations of motion for the two unknown dynamical variables $x, \theta$ and the undetermined Lagrange multiplier $\lambda$. Solve these equations for the same initial conditions as in (a) and determine the static frictional force of constraint between the hoop and the incline.

Solution: