A tiny particle of mass \( m \) slides without friction down a spherical surface of radius \( R \). The particle starts at the top with negligible speed.

(a) Determine the Lagrangian in polar coordinates, \( L(r, \theta, \dot{r}, \dot{\theta}) \), and the holonomic constraint \( f(r, \theta) = 0 \) of the sliding motion for as long as it lasts.

(b) Use the results of (a) and the conservation of energy to determine the force of constraint (normal force) during the sliding part of the motion.

(c) Determine the angle at which the particle leaves the sphere from the criterion that the force of constraint vanishes.

Solution: