[mex37] Noether's theorem III: pure Galilei transformation

Consider the Lagrangian $L(z,\dot{z}) = \frac{1}{2}m\dot{z}^2 - mgz$ of a particle with mass m moving vertically in 3D space under the influence of a uniform gravitational field. Show that the transformation $X = x,\ Y = y,\ Z = z + \epsilon t$ is a symmetry transformation by establishing the relation

$$L'(Z,\dot{Z},t,\epsilon) = L(Z,\dot{Z}) + \frac{d}{dt}F(Z,t,\epsilon).$$

Find the function $F(Z,t,\epsilon)$ and the conserved quantity $I(z,\dot{z},t)$ associated with this symmetry.

Solution: