A particle of mass $m$ moves in the Yukawa potential $V(r) = -(k/r)e^{-r/\rho}$, where $k$ is a measure for the strength and $\rho$ a measure for the range of the interaction. Circular orbits exist only if the angular momentum $\ell$ does not exceed a certain value $\ell_{\text{max}}$. For any value $\ell < \ell_{\text{max}}$, there exist two circular orbits, one stable orbit at radius $R_S(\ell)$ and one unstable orbit at radius $R_U(\ell)$.

(a) Establish the (dimensionless) relation, $\ell/\sqrt{mk\rho} = f(R/\rho)$ which determines the values $R_U$ and $R_S$ for given $\ell < \ell_{\text{max}}$ and produce a sketch of that function.

(b) Determine the value of the (conveniently scaled) maximum angular momentum, $\ell_{\text{max}}/\sqrt{mk\rho}$.

(c) Determine the value $R_S(\ell_{\text{max}})/\rho = R_U(\ell_{\text{max}})/\rho$ of the (merged) scaled radii at maximum angular momentum.

Solution: