[mex94] Hamiltonian system specified by noncanonical variables

A classical dynamical system is specified by the following Hamilton's equations of motion for three noncanonical variables A, B, C:

$$\frac{d}{dt}A = -2BC, \quad \frac{d}{dt}B = -2AC, \quad \frac{d}{dt}C = 4AB.$$

The three variables satisfy the mutual Poisson brackets $\{A, B\} = C$, $\{B, C\} = A$, $\{C, A\} = B$.

(a) Determine the energy function $\bar{H}(A, B, C)$ of this system.

(b) Show that the function $I(A, B, C) = \sqrt{A^2 + B^2 + C^2}$ is an integral of the motion.

(c) Show that $q = \arctan(B/A)$, $p = C$ are a pair of canonical coordinates.

Solution: