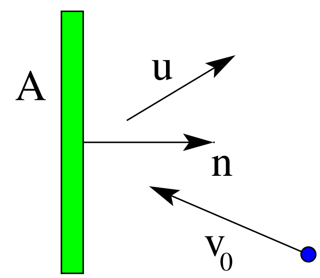
Kinetic forces and mobility [tln43]

Consider a *single-velocity beam*: a shower of particles with mass m, all with velocity $\vec{v}_0$, distributed randomly in space with particle density n_0.

Situation #1: A hard wall of area A and normal unit vector $\vec{n}$ moves with velocity $\vec{u}$ through the path of the single-velocity beam. Find the kinetic force experienced by the wall.



Rate of collisions (viewed from the rest frame of the wall):

$$\frac{dN}{dt} = -n_0 A\left[\vec{n}\cdot(\vec{v}_0-\vec{u})\right] \quad \text{if} \quad \vec{n}\cdot(\vec{v}_0-\vec{u}) < 0.$$

Momentum transfer per collision: $\Delta\vec{P} = 2m\vec{n}\left[\vec{n}\cdot(\vec{v}_0-\vec{u})\right].$

Kinetic force: $\vec{F} = \dfrac{dN}{dt}\Delta\vec{P} = -2mn_0 A\vec{n}\left[\vec{n}\cdot(\vec{v}_0-\vec{u})\right]^2.$

Situation #2: Consider a heavy hard sphere of radius R moving with velocity $\vec{u}$ in the path of a single-velocity beam of light particles (mass m, velocity $\vec{v}_0$, density n_0). The kinetic force experienced by the sphere is calculated in exercise [tex68]:

$$\vec{F} = \pi m n_0 R^2 |\vec{v}_0-\vec{u}|(\vec{v}_0-\vec{u}).$$

Situation #3: The mobility constant μ in the equation $\vec{u} = \mu\vec{F}_{app}$ relates the steady state velocity $\vec{u}$ of an object moving through a fluid to the external force applied to the object. In steady-state motion, the external force is balanced by the kinetic force $\vec{F}$ exerted by the fluid particles on the object: $\vec{F}_{app} = -\vec{F}$. The kinetic force exerted by a dilute gas (density n, particle mass m, temperature T) on a slowly moving heavy hard sphere (radius R, velocity $\vec{u}$ with $u \ll \langle v\rangle$) is calculated in exercise [tex69]:

$$\vec{F} = -\frac{8}{3}\sqrt{2\pi m k_B T}\,R^2 n\vec{u}.$$