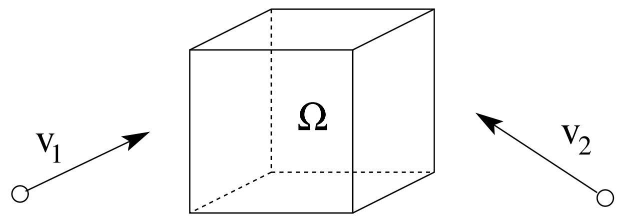
Collision rate and mean free path [tln44]

Consider two single-velocity beams of hard spheres with diameter d, mass m, particle densities n_1, n_2, and velocities $\vec{v}_1, \vec{v}_2$.

Find the collision rate of particles in a region of volume Ω at the intersection of the two beams.



View from rest frame of beam 1.

Number of particles in Ω: $N_1 = n_1\Omega,\ N_2 = n_2\Omega$.

Volume swept by one particle 2 inside Ω in time dt: $\omega_2 = \pi d^2|\vec{v}_2 - \vec{v}_1|dt$.

Volume swept by all particles inside Ω: $\Omega_2 = N_2\omega_2$.

Number of particles 1 inside Ω that will be hit in time dt: $dN = n_1\Omega_2 dt$.

Collision rate: $R_{coll} = \dfrac{dN}{dt} = n_1 n_2 \Omega \pi d^2 |\vec{v}_2 - \vec{v}_1|$.

Collision rate in classical ideal gas:
From the above result, the rate of particle collisions within a region Ω of a classical ideal gas with density n in thermal equilibrium at temperature T is then calculated in exercise [tex70]:

$$R = 2\Omega d^2 n^2 \sqrt{\pi k_B T/m}.$$

Mean free path of particle in classical ideal gas:
From the collision rate of particles in a classical ideal gas, the the mean free path (average distance travelled between collisons) is then calculated in exercise [tex71]:

$$\ell = \frac{1}{\sqrt{2}\pi d^2 n}.$$