Isothermal and adiabatic processes [tln8]

Fluid system:

Start from $TdS = dU + pdV$ with $U = U(T, V)$

$$TdS = \left(\frac{\partial U}{\partial T}\right)_V dT + \left[\left(\frac{\partial U}{\partial V}\right)_T + p\right] dV \overset{(a)}{=} C_V dT + \frac{1}{\alpha_p V}(C_p - C_V)dV$$

Isotherm: $dT = 0 \;\Rightarrow\; TdS = \frac{1}{\alpha_p V}(C_p - C_V)dV \overset{(b)}{=} -\frac{\kappa_T}{\alpha_p}(C_p - C_V)dp$

Adiabate: $dS = 0 \;\Rightarrow\; dT = -\frac{1}{\alpha_p V}\frac{C_p - C_V}{C_V}dV \overset{(c)}{=} \frac{\kappa_S}{\alpha_p}\frac{C_p - C_V}{C_V}dp$

(a) Use $C_p - C_V = \left[\left(\frac{\partial U}{\partial V}\right)_T + p\right]\left(\frac{\partial V}{\partial T}\right)_p, \quad \left(\frac{\partial V}{\partial T}\right)_p = V\alpha_p$

(b) Use $dV = \left(\frac{\partial V}{\partial p}\right)_T dp = -V\kappa_T dp$

(c) Use $dV = -V\kappa_S dp$

Magnetic system:

Start from $TdS = dU - HdM$ with $U = U(T, M)$.

$$TdS = \left(\frac{\partial U}{\partial T}\right)_M dT + \left[\left(\frac{\partial U}{\partial M}\right)_T - H\right] dM = C_M dT - \frac{1}{\alpha_H}(C_H - C_M)dM$$

Isotherm: $dT = 0 \;\Rightarrow\; TdS = -\frac{1}{\alpha_H}(C_H - C_M)dM = -\frac{\chi_T}{\alpha_H}(C_H - C_M)dH$

Adiabate: $dS = 0 \;\Rightarrow\; dT = \frac{1}{\alpha_H}\frac{C_H - C_M}{C_M}dM = \frac{\chi_S}{\alpha_H}\frac{C_H - C_M}{C_M}dH$