Average occupation numbers for MB, FD, and BE gases

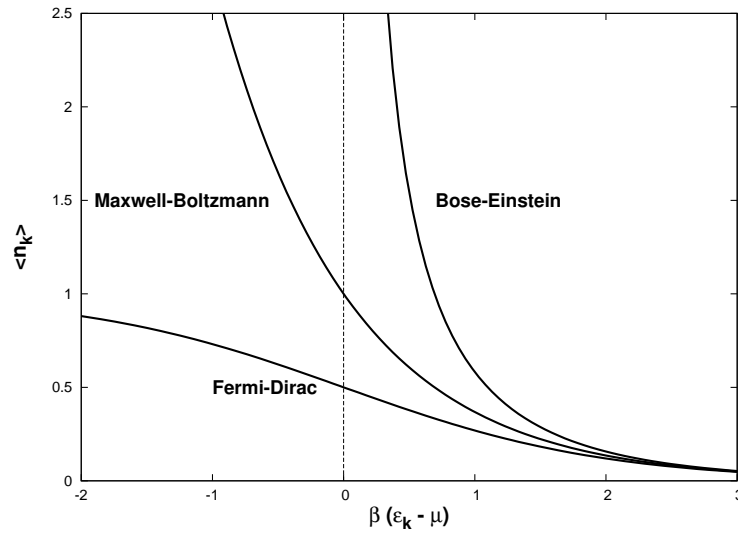Average occupation number of energy level ϵ_k:

$$\langle n_k \rangle = \frac{1}{e^{\beta(\epsilon_k - \mu)} + a}$$

- $a = 1$: Fermi-Dirac gas,
- $a = 0$: Maxwell-Boltzmann gas,
- $a = -1$: Bose-Einstein gas.

Range of 1-particle energies: $\epsilon_k \geq 0$.

BE gas restriction: $\mu \leq 0 \quad \Rightarrow \quad 0 \leq z \leq 1$.

The BE and FD gases are well approximated by the MB gas provided the thermal wavelength $\lambda_T = \sqrt{h^2/2\pi m k_B T}$ is small compared to the average interparticle distance:

$$\beta(\epsilon_k - \mu) \gg 1 \quad \Rightarrow \quad -\beta\mu \gg 1 \quad \Rightarrow \quad z \ll 1.$$

$$[\text{tex94}] \text{ for } \mathcal{D} = 3: \quad \Rightarrow \quad \lambda_T \ll (V/\mathcal{N})^{1/3}.$$