[tex100] **Heat capacity of ideal Fermi–Dirac gas I**

The internal energy of the ideal Fermi-Dirac gas is given by the expression

$$U = \frac{3}{2}\mathcal{N}k_BT\frac{f_{5/2}(z)}{f_{3/2}(z)}, \qquad f_n(z) = \frac{1}{\Gamma(n)}\int_0^\infty \frac{dx\, x^{n-1}}{z^{-1}e^x + 1},$$

where $z = e^{\mu/k_BT}$ is the fugacity.

(a) Infer from this result the following expression for the heat capacity $C_V = (\partial U/\partial T)_{V\mathcal{N}}$:

$$\frac{C_V}{\mathcal{N}k_B} = \frac{15}{4}\frac{f_{5/2}(z)}{f_{3/2}(z)} - \frac{9}{4}\frac{f_{3/2}(z)}{f_{1/2}(z)}.$$

To accomplish this task, use the recursion relation $zf_n'(z) = f_{n-1}(z)$ for the Fermi-Dirac functions and use the derivative $\partial/\partial T$ of the result $f_{3/2}(z) = \mathcal{N}\lambda_T^3/gV$ to calculate any occurrence of $(\partial z/\partial T)_{V\mathcal{N}}$ in the derivation.

(b) Show that in the Maxwell-Boltzmann limit (high temperature and/or low particle density) the result $C_V = \frac{3}{2}\mathcal{N}k_B$ is recovered from this expression.