[tex102] **Internal energy of ideal Fermi–Dirac gas**

Given are the following expressions for the average number of particles, the average energy, the average occupation number at $T = 0$, and the density of states for an ideal Fermi gas:

$$\mathcal{N} = \sum_k \langle n_k \rangle, \quad U = \sum_k \langle n_k \rangle \epsilon_k,$$

$$\langle n_k \rangle = \Theta(\epsilon_F - \epsilon_k), \quad D(\epsilon) = g\frac{2\pi V}{h^3}(2m)^{3/2}\epsilon^{1/2}.$$

Derive from these expressions

(a) the result $U_0 = \frac{3}{5}\mathcal{N}\epsilon_F$ for the internal energy at $T = 0$;

(b) the relation $U/V = (3/5)(\hbar^2/2m)(6\pi^2/g)(\mathcal{N}/V)^{5/3}$ between the energy density and the particle density.