[tex103] **Energy fluctuations and thermal response functions**

(a) Show that the following relation holds between the energy fluctuations in the microscopic ensemble and the heat capacity of a system described by a microscopic Hamiltonian H:

$$\langle (H - \langle H \rangle)^2 \rangle = k_B T^2 C_V.$$

(b) Prove the following relation in a similar manner:

$$\langle (H - \langle H \rangle)^3 \rangle = k_B^2 \left[T^4 \left(\frac{\partial C_V}{\partial T} \right)_V + 2T^3 C_V \right].$$

(c) Determine the relative fluctuations as measured by the quantities $\langle (H - \langle H \rangle)^2 \rangle / \langle H \rangle^2$ and $\langle (H - \langle H \rangle)^3 \rangle / \langle H \rangle^3$ for the classical ideal gas with N atoms.