[tex104] Anharmonic oscillator and thermodynamic perturbation

Consider an array of N one-dimensional anharmonic oscillators,

$$H = \sum_{l=1}^{N} \left[\frac{p_l^2}{2m} + V(q_l) \right], \quad V(q) = cq^2 - gq^3 + fq^4.$$

(a) Evaluate the canonical partition function perturbatively by treating the quadratic term of V exactly and considering only the leading nonzero corrections of the cubic and the quartic terms.
(b) Show that the heat capacity in this approximation is $C/Nk_B = 1 + [15g^2/16c^3 - 3f/4c^2]k_B T$.
(c) Show that the mean displacement in this approximation is $\langle q \rangle = (3g/4c^2)k_B T$.

Solution: