[tex104] **Anharmonic oscillator and thermal expansion**

Consider an array of N one-dimensional anharmonic oscillators,

$$H = \sum_{l=1}^{N} \left[\frac{p_l^2}{2m} + V(q_l) \right], \quad V(q) = cq^2 - gq^3 + fq^4,$$

in the canonical ensemble.

(a) Evaluate the canonical partition function under the assumption that the mean square displacement is so small that the factor $e^{\beta(gq^3 - fq^4)}$ can be expanded to leading order in all quantities where it appears.

(b) Show that the heat capacity in this approximation is $C/Nk_B = 1 + [15g^2/16c^3 - 3f/4c^2]k_BT$.

(c) Show that the mean displacement in this approximation is $\langle q \rangle = (3g/4c^2)k_BT$.