[tex105] **Statistical mechanics of blackbody radiation**

Electromagnetic radiation inside a cavity is in thermal equilibrium with the walls at temperature T. This system can be described by a grandcanonical ensemble (with $\mu = 0$) of photons (massless bosonic particles) with energy $\epsilon = \hbar\omega$ and density of states $\bar{D}(\omega) = (V/\pi^2 c^3)\omega^2$.

(a) Show that the internal energy can be expressed in the form

$$U(T,V) = \sigma V T^4, \quad \sigma = \frac{\pi^2 k_B^4}{15\hbar^3 c^3}$$

as postulated in a previous thermodydnamics problem [tex23].

(b) Show that the equation of state can be expressed in the form $pV = \frac{1}{3}U(T,V)$ as was also postulated in [tex23].