Consider the two-step cycle for a classical ideal gas \[ pV = Nk_B T, \quad C_V = \alpha Nk_B, \quad \gamma \equiv C_p/C_V = (\alpha + 1)/\alpha \] as shown. The first step (A) is an adiabatic compression and the second step (B) an expansion along a straight line segment in the \((V,p)\)-plane.

(a) Show that the difference in internal energy \(\Delta U \equiv U_1 - U_2\) is determined by the expression
\[
\frac{\Delta U}{p_1 V_1} = \alpha \left[ 1 - \left( \frac{V_1}{V_2} \right)^{\gamma-1} \right].
\]

(b) Show that the heat transfer \(\delta Q\) between system and environment during a volume increase from \(V\) to \(V + \delta V\) along the straight line segment is given by the expression
\[
\frac{\delta Q}{p_1 V_1} = \left[ (1 + \alpha)(1 + \sigma) - (1 + 2\alpha)\sigma \right] \frac{V}{V_1} dV, \quad \sigma = \frac{1 - (V_1/V_2)^\gamma}{V_2/V_1 - 1}.
\]

(c) Show that along the straight-line segment the system absorbs heat if \(V_1 < V < V_c\) and expels heat if \(V_c < V < V_2\), where \(V_c/V_1 = [(1 + \alpha)(1 + \sigma)]/[(1 + 2\alpha)\sigma]\).

Solution: