[tex11] How not to modify the ideal gas equation of state

Suppose we know empirically that for a real system with fixed n, the product of the pressure p and the volume V is a function of the temperature alone, $pV = f(T)$, and that the internal energy is also a function of the temperature alone, $U = U(T)$. These properties are realized in the classical ideal gas, where $f(T) = nRT$ and $U(T) = C_V T$ with $C_V = \alpha nR = $ const. Show that the only function $f(T)$ in the equation of state of the above form which is compatible with $U = U(T)$, i.e. with $(\partial U/\partial V)_T = 0$ is linear in T with zero intercept: $f(T) = rT$ with $r = $ const.

Solution: