BE gas in \( \mathcal{D} \) dimensions II: isochore

(a) From the fundamental thermodynamic relations for the Bose-Einstein gas in \( \mathcal{D} \) dimensions (see [tsc14]), derive the following parametric expression for the isochore at \( T \geq T_c \):

\[
\frac{p}{p_v} = \frac{g_{\mathcal{D}/2+1}(z)}{\left[ g_{\mathcal{D}/2}(z) \right]^{2/\mathcal{D}}}, \quad \frac{T}{T_v} = \left[ g_{\mathcal{D}/2}(z) \right]^{-2/\mathcal{D}},
\]

where \( k_B T_v = \Lambda v^{-2/\mathcal{D}} \) and \( p_v = \Lambda v^{-2/\mathcal{D}+1} \) with \( \Lambda \equiv h^2/(2\pi m) \) are convenient reference values.

(b) Calculate the leading correction to the Maxwell-Boltzmann result at high temperature. (c) Calculate the exact dependence of \( p/p_v \) on \( T/T_v \) at \( T \leq T_c \) in \( \mathcal{D} > 2 \). Show that this result also holds asymptotically for \( T \ll T_v \) in dimensions \( \mathcal{D} = 1 \) and \( \mathcal{D} = 2 \).

Solution: