[tex119] FD gas in \(D\) dimensions: statistical interaction pressure

Consider the isochore of an ideal Fermi-Dirac gas in \(D\) dimensions, as given by the parametric relation (derived in [tex114] for bosons in analogy),

\[
\frac{p}{p_v} = \frac{T}{T_v} \frac{f_{D/2+1}(z)}{f_{D/2}(z)}, \quad \frac{T}{T_v} = \left[f_{D/2}(z)\right]^{-2/D}.
\]

where \(k_B T_v = \Lambda/v^{2/D}\), \(p_v = k_B T/v\), \(\Lambda \equiv k^2/2\pi m\), \(v \equiv gV/N\). The upward deviation of this result from the Maxwell-Boltzmann result, \(p/p_v = T/T_v\), is a manifestation of repulsive statistical interaction between fermions. (a) Calculate the high-\(T\) asymptotic dependence of \(p/p_v\) on \(T/T_v\) including the leading correction to MB behavior. (b) Calculate the low-\(T\) limit of \(p/p_v\). (c) Calculate the low-\(T\) limit of \(p/p_F\), where \(T_F = T_v[\Gamma(D/2 + 1)]^{2/D}\) is the Fermi temperature and \(p_F = k_B T_F/v\) the associated reference pressure. (d) Compare the differently scaled statistical interaction pressures \(p/p_v\) and \(p/p_F\) at \(T = 0\) in the limit \(D \to \infty\).

Solution: