[tex127] Quantum paramagnet (microcanoncal ensemble)

Consider an array of N noninteracting localized magnetic dipole moments $\mathbf{m}_i$ produced by localized electron spins in a paramagnetic insulator. In the presence of a magnetic field $\mathbf{H}$ pointing in z-direction, the Hamiltonian of this system represents the Zeeman energy:

$$\mathcal{H} = -\sum_{i=1}^{N} \mathbf{m}_i \cdot \mathbf{H} = -H \sum_{i=1}^{N} m_i^z = -h \sum_{i=1}^{N} \sigma_i.$$

where $h = H/2$ and $\sigma_i = 2m_i^z = \pm 1$.
(a) Calculate the entropy $S(E, h, N)$ of this system in the *microcanonical* ensemble via saddle point method as follows. Express the number of distinct microstate at enthalpy E in the form $N_\Delta = \sum_{\sigma_1} \cdots \sum_{\sigma_N} \delta(E - \mathcal{H})$ with the δ-function replaced by its Fourier integral. Then use the asymptotic Laplace expression for the integral and evaluate it retaining only contributions that are significant in the thermodynamic limit.
(b) Calculate from $S(E, h, N)$ an explicit expression for the enthalpy $E(T, h, N)$ and derive from it the magnetization $M(T, h, N)$.

Solution: