Room heater: Electric radiator versus heat pump

A room is to be kept at temperature $T_H = 294\text{K}, (21^\circ\text{C})$. The outdoor temperature is $T_L$. Heat, which leaks through the windows and doors at the rate $\dot{Q}_{\text{leak}} = \gamma \Delta T$ (Fourier’s law), must be replaced by a room heater at the same rate. The electric radiator converts electric power $\dot{W}_{\text{el}}$ into heat $\dot{Q}_{\text{el}}$ with 100% efficiency. The electric heat pump uses the amount $\dot{W}_{\text{sup}}$ of electric power to drive a Carnot cycle in the reverse, which extracts heat $\dot{Q}_{\text{L}}$ at temperature $T_L$ from the exterior and converts it (reversibly) into heat $\dot{Q}_{\text{H}} = \dot{Q}_{\text{L}} + \dot{W}_{\text{hp}}$ at temperature $T_H$. In the relation $\dot{W}_{\text{hp}} = (1 - \lambda)\dot{W}_{\text{sup}}$, $\lambda$ represents the energy loss in the gears of the heat pump. Quantities with overbars denote energy transfers per time unit.

(a) Find $\dot{W}_{\text{el}}$ as a function of $\gamma, T_H, T_L$, and $\dot{W}_{\text{sup}}$ as a function of $\gamma, \lambda, T_H, T_L$.
(b) Plot $\dot{W}_{\text{el}}/\gamma$ and $\dot{W}_{\text{sup}}/\gamma$ versus $t_L \equiv T_L - 273\text{K}$ (measured in $^\circ\text{C}$) for fixed $T_H = 294\text{K}$ and $\lambda = 0.8\ (20\%\ \text{efficiency})$.
(c) Determine the range of $T_L$ for which the heat pump is more economical than the radiator.

Solution: