[tex13] **Room heater: Electric radiator versus heat pump**

A room is to be kept at an indoor temperature $T_H = 294\text{K}, (21°\text{C})$. The outdoor temperature is T_L. Heat, which leaks through the windows and doors at the rate $\bar{Q}_{leak} = \gamma \Delta T$, must be replaced by a room heater at the same rate (γ is the heat emission coefficient). The electric radiator converts electric power $\bar{W}_{el}$ into heat with 100% efficiency.

The electric heat pump uses the amount $\bar{W}_{sup}$ of electric power to drive (in an idealized setting) a Carnot cycle in the reverse, which extracts heat $\bar{Q}_L$ at the outdoor temperature T_L from the exterior and converts it (reversibly) into heat $\bar{Q}_H = \bar{Q}_L + \bar{W}_{hp}$ at the indoor temperature T_H. In the relation $\bar{W}_{hp} = (1-\lambda)\bar{W}_{sup}$, λ represents the energy loss in the gears of the heat pump. In this exercise, all quantitites with overbars indicate energy transfers per time unit.

(a) Find $\bar{W}_{el}$ as a function of γ, T_H, T_L, and $\bar{W}_{sup}$ as a function of $\gamma, \lambda, T_H, T_L$.

(b) Plot $\bar{W}_{el}/\gamma$ and $\bar{W}_{sup}/\gamma$ versus $t_L \equiv T_L - 273\text{K}$ (measured in °C) for fixed $T_H = 294\text{K}$ and $\lambda = 0.8$ (80% loss).

(c) Determine the outdoor temperature range where the heat pump is more economical than the electric radiator.