[tex22] **Thermodynamics of a paramagnetic gas**

Consider the fixed amount $n = 1$mol of a paramagnetic gas, specified by the equations of state $pV = RT$, $M = H/T$, and by the heat capacity $C_{VM} = 3R/2$.

(a) Show that the internal energy is independent of V, M when expressed as a function of T: $U(T, V, M) = U_0 + \frac{3}{2}RT$.

(b) Show that the heat absorbed by the gas in an isothermal process $(T, p_0, H_0) \to (T, p_1, H_1)$ is $\Delta Q = RT \ln(p_0/p_1) + (H_0^2 - H_1^2)/2T$. For $H =$ const the heat is converted into expansion work and for $p =$ const into demagnetization work.

(c) Show that in an adiabatic process $(T_0, p_0, H_0) \to (T_1, p_1, H_1)$ the following relation holds between the intensive variables:

$$\frac{p_0}{p_1}\left(\frac{T_1}{T_0}\right)^{5/2} \exp\left(\frac{1}{2R}\left[\frac{H_0^2}{T_0^2} - \frac{H_1^2}{T_1^2}\right]\right) = 1.$$

For $T_1 = T_0$ the process is *adiabatic* and *isothermal*. In this case compression work is converted into demagnetization work.