[tex23] **Thermodynamics of black-body radiation**

Electromagnetic radiation inside a cavity is in thermal equilibrium with the walls at temperature T. The radiation has an energy density that depends only on the temperature, i.e. its internal energy has the form $U(T,V) = Ve(T)$. The radiation pressure is determined by the energy density alone: $p = \frac{1}{3}e(T)$.

(a) Use the consistency equations for the total differential dS to show that the energy density has the form $e(T) = \sigma T^4$, where σ is a constant, now known as the *Stefan-Boltzmann constant*. In this argument, the additional assumption enters that $e(T) \to 0$ for $T \to 0$.

(b) Determine the entropy $S(T,V)$ and the thermodynamic potentials $U(S,V)$, $E(S,p)$, $A(T,V)$, $G(T,p)$.

(c) Determine the isotherms and adiabates in the (V,p)-plane.

(d) Determine the response functions $C_V, C_p, \kappa_T, \kappa_S, \alpha_p$.