[tex58] Maxwell's velocity distribution (Boltzmann's derivation)

The velocity distribution $f(\mathbf{v})$ is guaranteed to be a stationary solution of the Boltzmann equation if it satisfies the equation $f(\mathbf{v}_1)f(\mathbf{v}_2) = f(\mathbf{v}'_1)f(\mathbf{v}'_2)$, where $\mathbf{v}_1, \mathbf{v}_2$ and $\mathbf{v}'_1, \mathbf{v}'_2$ are the velocities before and after an elastic pair collison. Elasticity means that the four quantities $p_x = m(v_{1x} + v_{2x}), p_y = m(v_{1y} + v_{2y}), p_z = m(v_{1z} + v_{2z}), E = \frac{1}{2}m(v_1^2 + v_2^2)$ are conserved by the collision.
Boltzmann uses the following arguments: (i) The absence of any further conservation laws implies that $f(\mathbf{v}_1)f(\mathbf{v}_2) = F(p_x, p_y, p_z, E)$; (ii) in the relation $\ln f(\mathbf{v}_1) + \ln f(\mathbf{v}_2) = \ln F(p_x, p_y, p_z, E)$ the additivity of the two functions on the left-hand side implies that $\ln F$ is a linear function of its variables: $\ln F(p_x, p_y, p_z, E) = a_1 p_x + a_2 p_y + a_3 p_z + a_4 E + a_5$.
Show that if the five coefficients $a_1, \ldots, a_5$ are determined such as to satisfy the requirements $\int d^3v f(\mathbf{v}) = 1$ (normalization), $\langle \mathbf{v} \rangle = 0$ (symmetry), and $\frac{1}{2}m\langle v^2 \rangle = \frac{3}{2}k_B T$ (equipartition), then $f(\mathbf{v})$ is the Maxwell distribution.

Solution: