Toward thermal equilibrium via particle transfer

A vessel with insulating walls is divided into two compartments by an internal wall that is also insulating, but has a small hole of area $A$. The two compartments contain dilute gases of slightly different densities, $n_{\pm} = n \pm \frac{1}{2} \Delta n$, at slightly different temperatures, $T_{\pm} = T \pm \frac{1}{2} \Delta T$. We set $\Delta n > 0$ and allow $\Delta T$ to be positive or negative.

(a) Show that the rates at which particles and energy are transferred through the hole are (in leading orders of $\Delta n$ and $\Delta T$):

$$\frac{dN}{dt} = \frac{dN^-}{dt} - \frac{dN^+}{dt} = \frac{A}{\sqrt{2\pi m}} \left[ \sqrt{k_B T} \Delta n + \frac{1}{2} \frac{n}{\sqrt{k_B T}} (k_B \Delta T) \right],$$

$$\frac{dE}{dt} = \frac{dE^-}{dt} - \frac{dE^+}{dt} = \frac{A\sqrt{2}}{\sqrt{\pi m}} \left[ (k_B T)^{3/2} \Delta n + \frac{3}{2} n\sqrt{k_B T} (k_B \Delta T) \right].$$

(b) If the compartment with the higher particle density is at the lower temperature, i.e. for $\Delta T < 0$, it is possible to create situations where either the particle flow or the energy flow is instantaneously zero. Find the values of $\Delta T / \Delta n$ in terms of $n$ and $T$ for which we have either $dN/dt = 0$ or $dE/dt = 0$ initially when the hole is opened.

Solution: