[tex67] **Rate of chemical reaction $A + A \to 2A$ in gas phase**

The rate at which a chemical reaction of the type $A + A \to 2A$ takes place in a dilute gas is $R = \alpha N_R$, where α is a constant and N_R is the density of pairs of atoms with a center-of-mass kinetic energy K_{cm} in excess of some value ϵ_0. Here K_{cm} is defined as the kinetic energy of the two particles in a reference frame that moves with the center-of-mass velocity.

Show that

$$R = \frac{2\alpha n^2}{\sqrt{\pi}} \int_{x_0}^{\infty} dx\, x^2 e^{-x^2}, \qquad x_0 = \sqrt{\epsilon_0/k_B T},$$

where n is the particle density.