[tex73] **Classical ideal gas (microcanonical ensemble)**

Consider a classical ideal gas of N atoms confined to an insulating box of volume V. The Hamiltonian of the system reflects the kinetic energy of $3N$ noninteracting degrees of freedom:

$$H = \sum_{i=1}^{3N} \frac{p_i^2}{2m}.$$

(a) Calculate the entropy $S(U, V, N)$ in the *microcanonical* ensemble. Show that the result is the Sackur-Tetrode equation:

$$S(U, V, N) = \frac{5}{2} N k_B + N k_B \ln \left[\frac{V}{N h^3} \left(\frac{4\pi m U}{3N} \right)^{3/2} \right].$$

(b) Derive the internal energy $U = \frac{3}{2} N k_B T$ and the equation of state $pV = N k_B T$ from $S(U, V, N)$.