Array of classical harmonic oscillators (microcanonical ensemble)

Consider an array of $N$ 3-dimensional classical harmonic oscillators, representing a system of $3N$ uncoupled degrees of freedom:

$$H = \sum_{i=1}^{3N} \left( \frac{p_i^2}{2m} + \frac{1}{2} m \omega_i^2 q_i^2 \right).$$

(a) Calculate the entropy $S(U, N)$ of this system in the microcanonical ensemble.  
(b) Derive the internal energy $U(T, N)$, and the heat capacity $C = \left( \partial U / \partial T \right)_N$.  

Solution: