[tex75] **Array of quantum harmonic oscillators (microcanonical ensemble)**

Consider an array of N 3-dimensional quantum harmonic oscillators:

$$H = \sum_{i=1}^{3N} \left[\hbar\omega \left(n_i + \frac{1}{2} \right) \right], \quad n_i = 0, 1, 2, \ldots .$$

(a) Calculate the entropy $S(U, N)$ of this system in the *microcanonical* ensemble.

(b) Derive the internal energy $U(T, N)$, and the heat capacity $C = (\partial U/\partial T)_N$ from $S(U, N)$.

(c) Plot C versus T for $0 \leq k_B T \leq 3\hbar\omega$.