[tex77] **Ultrarelativistic classical ideal gas (canonical ensemble)**

Consider a classical ideal gas of N atoms confined to a box of volume V in thermal equilibrium with a heat reservoir at an extremely high temperature T. The Hamiltonian of the system,

$$H = \sum_{l=1}^{N} |\mathbf{p}_l| c,$$

where c is the speed of light, reflects the ultrarelativistic energy of N noninteracting particles:

(a) Calculate the canonical partition function Z_N of this system.

(b) Derive from Z_N the Helmholtz free energy $A(T, V, N)$, the entropy $S(T, V, N)$, the pressure $p(T, V, N)$, the internal energy $U(T, N)$, and the chemical potential $\mu(T, V)$.

(c) Show that the pressure is equal to one third of the energy density and that the adiabates satisfy $p^3 V^4 =$ const.