[tex82] **Array of quantum harmonic oscillators (canonical ensemble)**

Consider an array of N 3-dimensional quantum harmonic oscillators:

$$H = \sum_{i=1}^{3N} \left[\hbar\omega \left(n_i + \frac{1}{2} \right) \right], \quad n_i = 0, 1, 2, \ldots .$$

(a) Calculate the canonical partition function Z_N for this model.

(b) Derive from Z_N the Helmholtz free energy $A(T, N)$, the internal energy $U(T, N)$, and the heat capacity $C = (\partial U/\partial T)_N$.

(c) Show that $U(T, N)$ approaches the result of [tex78] for the classical oscillators.

(d) Calculate the quantity $\langle n_i \rangle$ for a single degree of freedom. It reflects the average number of elementary energy quanta that are excited in one oscillator when it is in thermal equilibrium at temperature T.