[tex85] **Quantum paramagnet (two–level system)**

Consider an array of N noninteracting localized magnetic dipole moments $\mathbf{m}_i$ produced by localized electron spins in a paramagnetic insulator. In the presence of a magnetic field $\mathbf{H}$ pointing in z-direction, the Hamiltonian of this system represents the Zeeman energy:

$$\mathcal{H} = -\sum_{i=1}^{N} \mathbf{m}_i \cdot \mathbf{H} = -H \sum_{i=1}^{N} m_i^z, \quad m_i^z = \pm\frac{1}{2}.$$

(a) Calculate the canonical partition function Z_N of this system.

(b) Calculate the Gibbs free energy $G(T,H,N)$, the magnetization $M(T,H,N)$, the isothermal susceptibility $\chi_T(T,H,N)$, and the heat capacity $C_H(T,H,N)$.

(c) Plot M/N versus H for three values of T. Plot C_H/N versus T for three values of H.

(d) Show that the leading term in an expansion of χ_T at small H is H-independent and represents Curie's law $\chi_T \simeq N/4k_BT$.