[tex86] **Quantum paramagnet (Brillouin function)**

Consider an array of N noninteracting localized magnetic dipole moments $\mathbf{m}_i$ produced by localized effective atomic spins in a paramagnetic insulator. In the presence of a magnetic field $\mathbf{H}$ pointing in z-direction, the Hamiltonian of this system represents the Zeeman energy:

$$\mathcal{H} = -\sum_{i=1}^{N} \mathbf{m}_i \cdot \mathbf{H} = -H \sum_{i=1}^{N} m_i^z,$$

where m_i^z can assume the $2s+1$ values $(-s, -s+1, \ldots, s-1, s)$ for fixed $s = \frac{1}{2}, 1, \frac{3}{2}, \ldots$.

(a) Calculate the canonical partition function Z_N of this system.

(b) Calculate the Gibbs free energy $G(T,H,N)$. Calculate the magnetization $M(T,H,N)$ (Brillouin function).

(c) Set $s = \frac{1}{2}$ to recover the result of [tex85]. Take the limit $s \to \infty$ and recover the result of [tex84] for the rescaled quantities $\tilde{M} = M/s, \tilde{H} = Hs$.