[tex89] **Quantum rotational heat capacity of a gas at low temperature**

The rotational spectrum of two-atomic molecules consists of energy levels

$$E_{lm} = \frac{l(l+1)\hbar^2}{2I}; \quad l = 0, 1, 2, \ldots; \quad m = -l, -l+1, \ldots, l.$$

Show that the leading term of the rotational heat capacity of a gas of N molecules at low temperature ($T \ll \Theta$) has the form

$$C_R(T) \simeq 12 N k_B \left(\frac{\Theta}{T}\right)^2 e^{-2\Theta/T}, \quad \Theta = \frac{\hbar^2}{2Ik_B}.$$